ROUND I: Arirthmetic

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What is 10% of 0.01 divided by 5% of 0.001?

2. Find the answer when you multiply the medians of A and B and then subtract the product of the mode of C and the mean of D for these data sets.

5	4	97
6	95	99
3	98	96

A

77	8	75
9	76	10
79	7	74

B

72	74	75
76	72	74
77	74	73

C

10	4	5
8	6	5
1	3	3

D

3. How many integers between 1 and 100,000 are divisible by all of 2, 3, 4, 5, 6, 7, and 11?

ANSWERS

(! pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

Algonquin, Quaboag, Southbridge

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $\frac{-3w-11c}{c+15w} = -2$, find the value of $\frac{w}{c}$.

2. One rajah says to another," If you will give me one camel, then we will have an equal number of camels". The other replies, "No, if you give me one camel, then I will have double the number you have". How many does each have?

3. You cut a 3 x 3 square from a calendar page. If the sum of the 9 dates is divisible by 13, what is the number in the lower left corner of the square?

ANSWERS

(1 pt) 1. $\frac{w}{c} =$ _____

(2 pts) 2. _____ and _____

(3 pts) 3. _____

Bromfield, Burncoat, Quaboag

ROUND III: Set theory

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Between 1933 and 1995 there were 11 presidents of the United States and 14 vice-presidents. If 9 of the vice-presidents were never president, how many of the presidents were never vice-president?

2. Let $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 4, 5, 6, 7\}$, and $C = \{4, 5, 8, 9, 10\}$ and let the universe be $A \cup B \cup C$. List the elements in $[(A \cap B) \cup (B \cap C)]' \cap (A \cup B)'$. S' is the complement of set S

3. 100 students attended at least one of 3 concerrts, Pep Band, Country Sizzle, and Blue Mood. 48 were at Pep Band, 36 at Country Sizzle, and 60 at Blue Mood. 12 heard Pep Band and Country Sizzle, 20 heard Country Sizzle and Blue Mood, and 16 heard Pep Band and Blue Mood. How many attended all 3 concerts?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. { _____ }

(3 pts) 3. _____

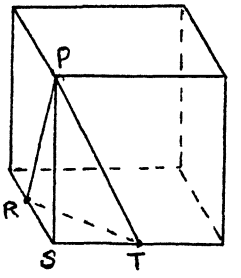
Burncoat, Shepherd Hill

ROUND IV: Measurement

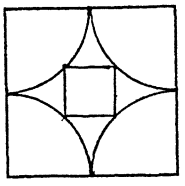
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS SPECIFIED IN THE PROBLEM

1. The scale on a map states that 1 cm represents 6 miles. How many square miles would be represented by an area on the map of 240 square cm?

2. T and R are the midpoints of two adjacent edges of the cube. If the volune of the cube is 1008, what is the volume of pyramid PRST?



3. In the diagram there are quarter circles centered at the vertices of a square with sides of length 2. There is a smaller square with sides parallel to the other square and vertices on the arcs. Find the area of the small square in a form involving a simplified radical or to the nearest .001.



ANSWERS

(1 pt) 1. _____ sq. mi

(2 pts) 2. _____

(3 pts) 3. _____

Algonquin, Burncoat, Westborough

ROUND V: Polynomial equations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Assume that a polynomial function p(x) of degree three has zeros -1, 5, and 7. State the zeros of 2p(x-3).

2. One root of the cubic equation $3x^3 - 22x^2 + 45x - 28 = 0$ is $x = 3 - \sqrt{2}$. What is the rational root of this equation?

3. What rational number k will make the factor x-k divide into $x^3 - 3x^2 + 5x - 12$ so that the remainder is k?

ANSWERS

(1 pt) 1. _____

(2 pts) 2. _____

(3 pts) 3. _____

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET 2 points each

1. 123 - 45 - 67 + 89 The expression to the left equals 100 and uses all the digits 1 through 9 exactly once in increasing order with only + or -. Write a similar expression using all the digits 1 through 9 exactly once in decreasing order with only + or - to equal 100.

2. Jane's house was built 10 years before Bob's house and 20 years after Ann's house. Twenty years ago the age of Ann's house was the same as the combined ages of Jane's and Bob's. What is the present age of Ann's house?

3. Consider the set { T, O, D, A, Y }. How many subsets can be made of two or more letters if at least one letter must be a vowel?

4. What are the lengths of the bases of an isosceles trapezoid in which the two equal length sides are 5, the perimeter is 34 , and the area is 48 ?

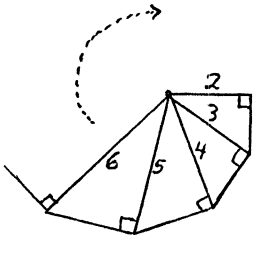
5. For what value(s) of the coefficient a do these equations have a real, common solution? $x^2 - ax + 1 = 0$ and $x^2 - x + a = 0$

6. Simplify $\dfrac{\sqrt{\sqrt{\sqrt{x^{16}}}}}{x}$.

7. Arrange the numbers 2^{333}, 3^{210}, and 5^{144} in increasing order.

8. If an angle is the union of two rays having the same endpoint, how many angles are formed by six noncollinear rays that have the same endpoint?

9. Continuing the pattern in the diagram to the right, how many right triangles can be made before overlapping begins? Include the four shown.



October 13, 1999 WOCOMAL Varsity Meet ANSWERS

ROUND I arith

1. 1 pt 20
2. 2 pts 6660
3. 3 pts 21

ROUND II Alg 1

1. 1 pt $\frac{1}{3}$
2. 2 pts 5 and 7 either order
3. 3 pts 19

ROUND III sets

1. 1 pt 6
2. 2 pts $\{9, 10\}$ either order
3. 3 pts 4

ROUND IV meas.

1. 1 pt 8640
2. 2 pts 42
3. 3 pts $6-4\sqrt{2}$ OR 0.343

ROUND V poly eq

1. 1 pt 2, 8, 10 any order
2. 2pts $\frac{4}{3}$ OR $1\frac{1}{3}$ OR $1.\overline{3}$
3. 3 pts 3

TEAM ROUND 2 pts each

1. $98-76+54+3+21$
2. 70 years (may omit)
3. 22
4. 9 and 15 either order
5. -2
6. x
7. $3^{210} < 2^{333} < 5^{144}$
8. 15
9. 11

Oct 13 1999 WOCOMAL Varsity Meet BRIEF SOLUTIONS

ROUND I

1. $\dfrac{(.10)(.01)}{(.05)(.001)} = \dfrac{.001}{.00005} = 20$

2. $(95)(74) - (74)(5) = 6660$

3 $100{,}000 \div$ LCM of $2,3,4,5,6,7,11$

$= 100{,}000 - 4620 = 21\,645$

$\therefore$ 21 integers

ROUND II

1 $-3w - 11c = -2c - 30w$

$27w = 9c \Rightarrow \dfrac{w}{c} = \dfrac{1}{3}$

2 one x, other: y

$\left\{\begin{matrix} x+1 = y-1 \\ 2(x-1) = y+1 \end{matrix}\right\} \Rightarrow x = 5,\ y = 7$

or trial and error with integers

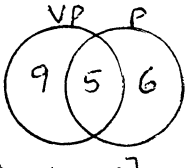
3.

$n-8$	$n-7$	$n-6$
$n-1$	n	$n+1$
$n+6$	$n+7$	$n+8$

For sum, $9n$, to be divisible by 13,
$n = 13$ and $n+6 = 19$

ROUND III

1.



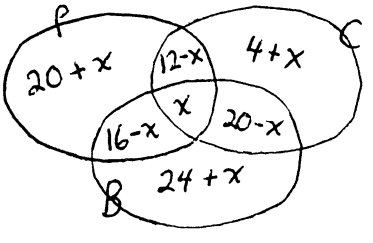
2 $[(A \cap B) \cup (B \cap C)] = \{4,6\} \cup \{4,5\} = \{4,5,6\}$

$[\quad]' = \{1,2,3,7,8,9,10\}$

$(A \cup B) = \{1,2,3,4,5,6,7,8\}$

$(\quad)' = \{9,10\} \longrightarrow$ Ans $\{9,10\}$

3.



SUM = 100

$96 + x = 100$

$x = 4$

ROUND IV

1 $1\ cm^2 = 36\ m^2$

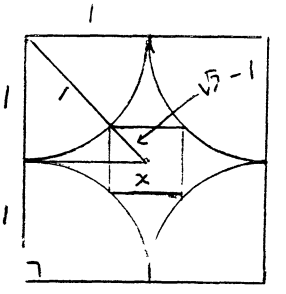
$240\ cm^2 = 240 \cdot 36 = 8640\ m^2$

2. If e = cube edge,

pyramid vol $= \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2}\,\frac{e}{2}\,\frac{e}{2}\right)e$

$= \dfrac{e^3}{24} = \dfrac{1008}{24} = 42$

3



45, 45, 90 $\triangle$s

$x = 2\left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)$

$= 2 - \sqrt{2}$

$x^2 = 6 - 4\sqrt{2}$

$= 0.343$

ROUND V

1 Add 3 to the given zeros to get 2, 8, and 10

2. $3+\sqrt{2}$ is also a root

Let r = 3rd root.

$(3+\sqrt{2})(3-\sqrt{2})r = \dfrac{28}{3}$

$7r = \dfrac{28}{3} \Rightarrow r = \dfrac{4}{3}$

3 synthetic subst

1	-3	5	-12	$\lfloor k$
	k	k^2-3k	k^3-3k^2+5k	
1	$k-3$	k^2-3k+5	$\underbrace{k^3-3k^2+5k-12}_{=k}$	

$k^3 - 3k^2 + 4k - 12 = 0$

$k^2(k-3) + 4(k-3) = 0$

$(k^2+4)(k-3) = 0 \Rightarrow k = 3$

TEAM ROUND

1. My trial and error found
$98 - 76 + 54 + 3 + 21 = 100$.
Apparently other answers are possible

2.

	Now	20 yrs ago
Jane's	A	A-20
Bob's	A-30	A-50
Ann's	A	A-20

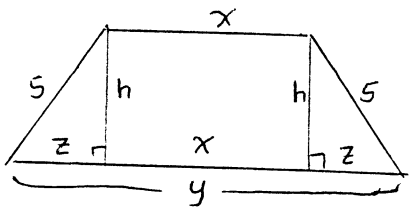
$A - 20 = 2A - 90 \Rightarrow A = 70$

3.

CT	OTD	OTDA
CD	OTA	OTDY
OA	OTY	ODAY
CY	ODA	OTAY
AT	ODY	ATDY
AD	OAY	
AY	ATO	OTDAY
	ATY	
	ADY	

22

4.



perim $x + y + 10 = 34 \Rightarrow x + y = 24$

area $\frac{1}{2}h(x+y) = 48 \Rightarrow 12h = 48$ and $h = 4$

rt $\triangle$, Pythag thm gets $z = 3$

$x + y = x + 6 + x = 24 \Rightarrow x = 9,\ y = 15$

5 Set $\frac{a \pm \sqrt{a^2 - 4}}{2} = \frac{1 \pm \sqrt{1 - 4a}}{2}$.

Note $a \geq 2$ or $a \leq -2$ and $a \leq \frac{1}{4}$ from $\sqrt{\ }$

With graphing calculator, graph these with X for a. One solution seems likely: X = a = 2 Check it

5 cont. $x^2 + 2x + 1 = 0$ $x = -1$ only

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\Leftrightarrow x = -1$

6 $\frac{\left(\left(\left(x^{16}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}{x} = \frac{x^2}{x} = x$

7. If your calculator will work with powers of 10 form with exponent ≥ 100, this is too easy (My TI-81 did not)

One way Use $\log a^x = x \log a$ and compare these expressions

Another way

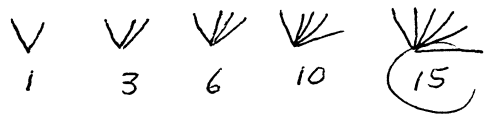
$2^{332} = 8.749 \times 10^{99}$ so $2^{333} \approx 17 \times 10^{99}$

$3^{209} = 5.228 \times 10^{99}$ so $3^{210} = 15 \times 10^{99}$

$5^{143} = 8.968 \times 10^{99}$ so $5^{144} \approx 45 \times 10^{99}$

8. $6C2 = 15$

OR



9. Consider angles at common vertex

		sum
1st = $\cos^{-1}\frac{2}{3}$	= 48.19°	
2nd = $\cos^{-1}\frac{3}{4}$	= 41.41°	89.60°
3rd = $\cos^{-1}\frac{4}{5}$	= 36.87°	126.47°
.	33.56°	160.03°
.	31.00°	191.03
.	28.96°	219.99°
	27.27°	247.26°
	25.84°	273.10°
	24.62°	297.72°
	23.56°	321.28°
11th = $\cos^{-1}\frac{12}{13}$ =	22.62°	348.90°
12th overlaps	21.79°	365.69°